(This page intentionally left blank)

11 Strength of the Boiler Shell

Water and steam under pressure exert force in all directions. Pressure does not care whether the boiler surface is horizontal, vertical, curved, or flat. At every point, the pressure acts perpendicular to the surface.

The pressure-retaining parts of a boiler are subjected to enormous forces. Understanding how those forces are carried through the structure is the key to understanding boiler strength.

11.1 Curved and Flat Surfaces

Pressure acts equally in all directions, but not all shapes respond to pressure in the same way.

A round surface wants to remain round. A cylindrical shell under pressure expands slightly, but the load is carried efficiently as tension around the shell.

A flat surface behaves differently. Pressure causes the plate to bend and bulge outward. As the plate becomes larger, the bending stresses increase rapidly. For this reason, flat surfaces usually require stays or reinforcement.

Because a cylindrical shell carries pressure so efficiently, it forms the main body of most boilers. The shell, barrel, steam dome, and many other pressure-retaining parts are based on curved shapes.

11.2 Why boilers don't fly

To understand why the cylindrical shell is such an efficient structure, it is useful to consider how the shell would fail if the internal pressure became excessive.

The simplest way to understand shell strength is to ask how a cylindrical shell would fail if the pressure became too high.

Imagine a cylindrical shell divided into two halves by a longitudinal plane. The pressure inside the boiler pushes upward on one half and downward on the other. The total separating force is the pressure multiplied by the projected area.

Consider the areas of the lower and upper portions of the boiler cylinder along a horizontal plane dividing the boiler in half. In Fig. 1, the cross section of the cylinder is shown in three stages. In the first stage, the cross section is circular. In the second stage, the lower portion of the cylinder has been partially flattened. In the third stage, the lower portion has been completely flattened, resulting in the figure shown in Fig. 2.

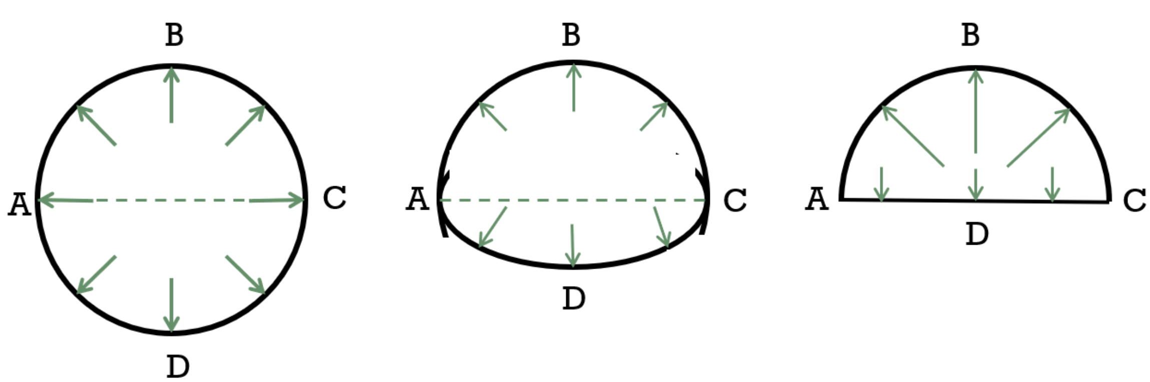

Fig. 1 Cylinder shapes

The cross-section projected along the shell looks like the upper half of a horizontal cylinder.

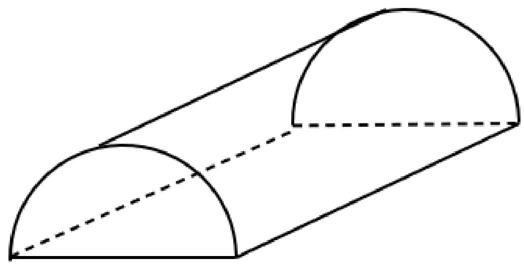

Fig. 2 Upper half of cylinder

The arrows in Fig. 1 show internal pressure as being normal to the inner shell. In the first stage, the sum of the pressure along the projection of semicircle ABC is the same as the the sum of the pressure along the projection of ADC. The pressures are balanced.

In the second stage, the lower shape ADC also experiences all the pressure below the horizontal line AC. This is true in the third stage as well.

It is clear that the length of the arc ADC in stage two is less than the arc ADC in stage one, and it is easy to see that the volume of the shape ADC in stage two is also less than the shape in stage one.

In stage three, the lower shape has been collapsed to a line, which has a volume of zero.

The question is now this: what is the combined force due to pressure along line ADC in each of the three stages, in comparison to the pressure along ABC.

Consider what must happen if the the lower force was less. The sum of the vertical components would have to be less on the lower shell than the upper shell. If this were the case, the boiler would rise due to the differential. Either we have invented a new form of air travel, or this cannot be the case.

In fact the upper and lower forces must match. This means the force on the flat lower surface in Fig. 2 must match that on the curved upper surface.

Now consider what must happen along the line where the two surfaces meet. The material along this line experiences two equal forces in opposite directions. As we will see later, when metal is pulled in two opposite directions, the force required to tear the metal is determined by the tensile strength of the material. The resisting area is the area of metal crossing the imaginary dividing plane. This area equals the shell thickness multiplied by the cylinder length at each side, giving a total resisting area of $2LT$.

For a cylinder with diameter $D = 30\,\mathrm{in}$ and length $L = 96\,\mathrm{in}$ with internal pressure $P = 100\,\frac{\mathrm{lb}}{\mathrm{in}^2}$, the total upward (or downward) force is

$$\begin{aligned} P_D &= (D \cdot L \cdot P) \\ &= \left(30\,\mathrm{in} \cdot 96\,\mathrm{in} \cdot 100\,\tfrac{\mathrm{lb}}{\mathrm{in}^2}\right) \\ &= 288000\,\mathrm{lb} \end{aligned}$$

If the cylinder has a thickness of $T = 0.250\,\mathrm{in}$, the area of a tear along the complete seam is

$$
\begin{aligned}
A_s &= (2LT) \\
&= (296\,\mathrm{in} 0.250\,\mathrm{in}) \\
&= 48\,\mathrm{in}^2
\end{aligned}
$$

So the force on the seam is

$$
\begin{aligned}
\frac{P_D}{A_s} &= \frac{P_D}{A_s} \\
&= 6000
\end{aligned}
$$

The shell will burst when the force on the seam equals the tensile strength of the material. If the tensile strength is $\sigma_t = 60000\,\frac{\mathrm{lb}}{\mathrm{in}^2}$

$$\frac{D \cdot L \cdot P_b}{2LT} = \sigma_t$$

Solving for P_b,

$$
\begin{aligned}
P_b &= \frac{2LT\sigma_t}{D \cdot L} \\
&= \frac{296\,\mathrm{in} 0.250\,\mathrm{in} \sigma_t}{30\,\mathrm{in} \cdot 96\,\mathrm{in}} \\
&= 1000
\end{aligned}
$$

That is, a cylinder 30 inches wide, 96 inches long and a quarter inch thick with a tensile strength of 60000 will burst at 1000 PSI internal pressure.

One more thing. Looking at the equation $P_b = \frac{2LT\sigma_t}{D \cdot L}$, note that length appears both in the numerator and the denominator of the pressure-area ratio. The equation can be reduced to $P_b = \frac{2T\sigma_t}{D}$. The reduced form shows that bursting pressure is a function of thickness and diameter. The cylinder could be an inch long or a mile long, and the bursting pressure will still be the same.

11.3 Safety Factor and MAWP

The previous section showed that a cylindrical boiler shell fails in tension when the hoop stress reaches the tensile strength of the material. The calculations showed the relatively thin shell would burst at 1000 psi.

A shell that bursts at 1000 psi should obviously not be operated at 1000 psi. To determine a safe maximum operating pressure, a **factor of safety** is applied.

This section shows how a factor of safety relates to the physical characteristics of metal.

11.3.1 Tensile Failure of a Bolt

Before looking at the formal definitions of stress, strain and elasticity, consider the bolt in Fig. 3 that has been pulled in tension until failure.

As the tensile force increased, the bolt first stretched elastically. During this stage the bolt lengthened slightly, but would have returned to its original shape if the load had been removed.

At a higher load, the material reached its yield strength and began to deform permanently. The deformation became concentrated in one region of the bolt, producing the narrowed section known as a *neck*.

As the neck became smaller, the remaining metal had to carry the entire load across a progressively smaller cross-sectional area. The stress in the neck therefore increased rapidly until the material could no longer support the load.

The bolt finally fractured at its ultimate tensile strength.

The necked shape seen on the failed specimen is characteristic of a *ductile* material. Before breaking, the material gave a visible warning by deforming significantly. A more brittle material would show little or no necking before fracture.

Boiler shells behave in a similar way. Internal pressure creates tensile forces in the shell. If these forces become large enough, the metal first yields and deforms, and only at a higher stress does it tear apart. For this reason boiler design is based not only on preventing rupture, but also on keeping working stresses well below the yield point.

Fig. 3 Tensile test of AlMgSi alloy

11.3.2 Stress, Strain, and Elasticity

A force acting on a material creates **stress**.

Stress causes the material to deform, producing **strain**.

At low stresses, most metals behave elastically. The deformation is completely recoverable when the load is removed.

As the load increases, the material reaches its **yield strength**. Beyond this point, permanent deformation occurs.

If the load continues to increase, the material eventually reaches its **ultimate tensile strength**, and finally fractures.

For most boiler materials, the yield strength is approximately one-half of the tensile strength.

A typical stress-strain curve therefore contains three important regions:

1. Elastic region
2. Yield point
3. Plastic deformation leading to fracture

Safe operation should remain well within the elastic region.

11.3.3 Why a Safety Factor is Needed

The tensile strength represents the point at which the material tears apart.

In practice, boilers are never operated anywhere near this limit.

A **factor of safety** provides a margin between the working stress and the failure stress.

The Maximum Allowable Working Pressure (MAWP) is therefore

$P_{MAWP} = \frac{P_B}{F}$ where F is the factor of safety.

Substituting the bursting-pressure equation gives

$P_{MAWP} = \frac{2T\sigma_t}{F \cdot D}$

11.3.4 Using a Factor of Safety of Four

For the example boiler,

$F = 4$

so

$P_{MAWP} = \frac{2 \times 0.25 \times 60{,}000}{4 \times 30} = 250 \text{ psi}$

Thus a boiler that theoretically bursts at 1000 psi is limited to a working pressure of 250 psi.

11.3.5 Relationship to Yield Strength

The reason a factor of safety of four works well can be understood from the stress-strain curve.

For typical boiler steels,

$\sigma_y = \frac{\sigma_t}{2}$

where σ_y is the yield strength.

A factor of safety of four limits the working stress to

$\frac{\sigma_t}{4}$

which is approximately

$\frac{\sigma_t}{2}$

In other words, the working stress lies in the lower half of the elastic range and well below the onset of permanent deformation.

This provides a practical margin for:

- pressure fluctuations,
- material variations,
- corrosion and wear,
- manufacturing tolerances,
- minor stress concentrations,
- uncertainties in operation.

The boiler therefore operates comfortably within the region where the metal will return to its original shape when the load is removed.

11.3.6 True Stress and Engineering Stress

When a tensile test is performed, the engineering stress is calculated using the original cross-sectional area of the specimen.

$s = \frac{F}{A_0}$

As the specimen stretches, its width and thickness decrease.

The actual stress in the material at any instant is therefore greater than the engineering stress. This is called **true stress**.

$\sigma = s \cdot (1 + e)$

where e is the engineering strain.

Similarly,

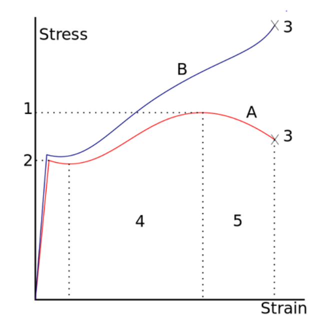

Fig. 4 Stress-Strain Curve

1. Ultimate strength
2. Yield strength

$$\varepsilon = \ln(1 + e)$$

where ε is the true strain.

Engineering stress is convenient because it requires only the original dimensions of the specimen. True stress better represents the actual state of the material during deformation.

3. Fracture
4. Strain hardening region
5. Necking

A. Engineering stress
B. True stress

For boiler design, however, the important point is that the allowable working stress is intentionally kept far below both ultimate tensile strength and fracture. A factor of safety of four places operation safely within the elastic region of the material.

11.3.7 Design Implications

The MAWP equation shows that

$$P_{MAWP} = \frac{2T\sigma_t}{F \cdot D}$$

A larger diameter requires either:

- a thicker shell,
- a stronger material,
- a lower working pressure,
- or a smaller factor of safety approved by code.

Conversely, increasing shell thickness increases the safe working pressure in direct proportion.

This simple relationship is one of the fundamental principles of boiler and pressure-vessel design.